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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MFP3 Further Pure 3

Mark Scheme

2008 examination - January series

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

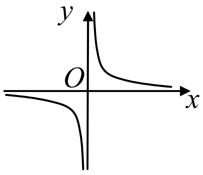
MFP3

Q	Solution	Marks	Total	Comments
1(a)	$y(2.1) = y(2) + 0.1[2^2 - 1^2]$	M1A1		
	$= 1+0.1\times3 = 1.3$	A1	3	
(b)	$y(2.2) = y(2) + 2(0.1)[f(2.1, y(2.1))]$	M1		
	$\ldots = 1+2(0.1)[2.1^2 - 1.3^2]$	A1√		Ft on cand's answer to (a)
	$\ldots = 1+0.2\times2.72 = 1.544$	A1	3	CAO
	Total		**6**	
2(a)	Area $= \frac{1}{2}\int(1+\tan\theta)^2\,\mathrm{d}\theta$	M1		Use of $\frac{1}{2}\int r^2\,\mathrm{d}\theta$
	$\ldots = \frac{1}{2}\int(1+2\tan\theta+\tan^2\theta)\,\mathrm{d}\theta$	B1		Correct expansion of $(1+\tan\theta)^2$
	$=\frac{1}{2}\int(\sec^2\theta+2\tan\theta)\,\mathrm{d}\theta$	M1		$1+\tan^2\theta=\sec^2\theta$ used
	$=\frac{1}{2}\Big[\tan\theta+2\ln(\sec\theta)\Big]_0^{\frac{\pi}{3}}$	A1√ B1√		Integrating $p\sec^2\theta$ correctly Integrating $q\tan\theta$ correctly
	$=\frac{1}{2}[(\sqrt{3}+2\ln2)-0]=\frac{\sqrt{3}}{2}+\ln2$	A1	6	Completion. AG CSO be convinced
(b)	$OP=1;\ \ OQ=1+\tan\frac{\pi}{3}$	B1		Both needed. Accept 2.73 for OQ
	Shaded area =			
	'answer (a)' $-\frac{1}{2}OP\times OQ\times\sin\left(\frac{\pi}{3}\right)$	M1		
	$=\frac{\sqrt{3}}{2}+\ln2-\frac{\sqrt{3}}{4}(1+\sqrt{3})$	A1	3	ACF. Condone 0.376… if exact 'value' for area of triangle seen
	$=\frac{\sqrt{3}}{4}+\ln2-\frac{3}{4}$			
	Total		**9**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$(m+2)^2 = -1$	M1		Completing sq or formula
	$m = -2 \pm \mathrm{i}$	A1		
	CF is $\mathrm{e}^{-2x}(A\cos x + B\sin x)$	M1		If m is real give M0
	{or $\mathrm{e}^{-x}A\cos(x+B)$ but not $A\mathrm{e}^{(-1+\mathrm{i})x} + B\mathrm{e}^{(-1-\mathrm{i})x}$ }	A1√		Ft on wrong a's and b's but roots must be complex
	PI try $y = p \Rightarrow 5p = 5$ PI is $y = 1$	B1		
	GS $y = \mathrm{e}^{-2x}(A\cos x + B\sin x) + 1$	B1√	6	Their CF + their PI with two arbitrary constants.
(b)	$x=0,\ y = 2 \Rightarrow A = 1$	B1√		Provided previous B1√ awarded
	$y'(x) = -2\mathrm{e}^{-2x}(A\cos x + B\sin x) +$	M1		Product rule used
	$+\ \mathrm{e}^{-2x}(-A\sin x + B\cos x)$	A1√		
	$y'(0) = 3 \Rightarrow 3 = -2A + B \Rightarrow B = 5$ $y = \mathrm{e}^{-2x}(\cos x + 5\sin x) + 1$	A1√	4	Ft on one slip
	Total		**10**	
4(a)	The interval of integration is infinite	E1	1	OE
(b)	$\int x\mathrm{e}^{-3x}\,\mathrm{d}x = -\frac{1}{3}x\mathrm{e}^{-3x} - \int -\frac{1}{3}\mathrm{e}^{-3x}\,\mathrm{d}x$	M1 A1		Reasonable attempt at parts
	$= -\frac{1}{3}x\mathrm{e}^{-3x} - \frac{1}{9}\mathrm{e}^{-3x}$ {+c}	A1√	3	Condone absence of $+c$
(c)	$I = \int_1^{\infty} x\mathrm{e}^{-3x}\,\mathrm{d}x = \lim_{a\to\infty} \int_1^{a} x\mathrm{e}^{-3x}\,\mathrm{d}x$			
	$\lim_{a\to\infty} \{-\frac{1}{3}a\mathrm{e}^{-3a} - \frac{1}{9}\mathrm{e}^{-3a}\} - \left[-\frac{4}{9}\mathrm{e}^{-3}\right]$	M1		F(a) – F(1) with an indication of limit '$a \to \infty$'
	$\lim_{a\to\infty} a\mathrm{e}^{-3a} = 0$	M1		For statement with limit/limiting process shown
	$I = \frac{4}{9}\mathrm{e}^{-3}$	A1	3	
	Total		**7**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
5	IF is $e^{\int \frac{4x}{x^2+1}\,dx}$	M1		
	$= e^{2\ln(x^2+1)}$	A1		
	$= e^{\ln(x^2+1)^2} = (x^2+1)^2$	A1√		Ft on $e^{p\ln(x^2+1)}$
	$\frac{d}{dx}\left(y(x^2+1)^2\right) = x(x^2+1)^2$	M1		LHS as d/dx(y×cand's IF) PI and also RHS of form $kx(x^2+1)^p$
		A1√		
	$y(x^2+1)^2 = \int x(x^2+1)^2\,dx$			
	$y(x^2+1)^2 = \frac{1}{6}(x^2+1)^3 + c$	M1 A1		Use of suitable substitution to find RHS or reaching $k(x^2+1)^3$ OE Condone missing c
	$y(0) = 1 \Rightarrow c = \frac{5}{6}$	m1		
	$y = \frac{1}{6}(x^2+1) + \frac{5}{6(x^2+1)^2}$	A1	9	Accept other forms of f(x) eg $y = \frac{\left(\frac{x^6}{6} + \frac{2x^4}{4} + \frac{x^2}{2} + 1\right)}{(x^2+1)^2}$
	Total		**9**	
6(a)	$r^2 2\sin\theta\cos\theta = 8$	M1		$\sin 2\theta = 2\sin\theta\cos\theta$ used
	$x = r\cos\theta \quad y = r\sin\theta$	M1		Either **one** stated or used
	$xy = 4$, $y = \frac{4}{x}$	A1	3	Either OE eg $y = \frac{8}{2x}$
(b)		B1	1	
(c)	$r = 2\sec\theta$ is $x = 2$	B1		
	Sub $x = 2$ in $xy = 4 \Rightarrow 2y = 4$ In cartesian, $A(2, 2)$	M1		
	$\Rightarrow \tan\theta = \frac{y}{x} = 1 \Rightarrow \theta = \frac{\pi}{4}$ $\Rightarrow r = \sqrt{x^2+y^2} = \sqrt{8}$	M1		Used either $\tan\theta = \frac{y}{x}$ or $r = \sqrt{x^2+y^2}$
	$\theta = \frac{\pi}{4}$; $r = \sqrt{8}$	A1	4	r must be given in surd form
	Altn2: Eliminating r to reach eqn. in $\cos\theta$ and $\sin\theta$ only (M1) $\theta = \frac{\pi}{4}$ (A1) Substitution $r=2\sec\left(\frac{\pi}{4}\right)$ (m1) $r = \sqrt{8}$ (A1) OE surd			Altn3: $r\sin\theta = 2$ (B1) Solving $r\cos\theta = 2$ and $r\sin\theta = 2$ simultaneously (M1) $\tan\theta = 1$ or $r^2=2^2+2^2$ (M1) $\theta = \frac{\pi}{4}$; $r = \sqrt{8}$ (A1) need both
	Total		**8**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$\ln(1+2x) = 2x - 2x^2 + \frac{8}{3}x^3 \ldots$	M1 A1	2	Use of expansion of $\ln(1+x)$ Simplified 'numerators'.
(ii)	$-\frac{1}{2} < x \le \frac{1}{2}$	B1	1	
(b)(i)	$y = \ln\cos x \Rightarrow y'(x) = \frac{1}{\cos x}(-\sin x)$	M1		
	$y''(x) = -\sec^2 x$ $y'''(x) = -2\sec x(\sec x \tan x)$	A1 M1		ACF Chain rule OE
	$\{y'''(x) = -2\tan x(\sec^2 x)\}$	A1√	4	Ft a slip…accept unsimplified
(ii)	$y''''(x) = -2[\sec^2 x(\sec^2 x) +$ $\tan x(2\sec x(\sec x \tan x))]$	M1 A1		Product rule OE ACF
	$y''''(0) = -2[(1)^2 + 0] = -2$	A1√	3	Ft a slip
(iii)	$\ln\cos x \approx 0 + 0 + \frac{x^2}{2}(-1) + 0 + \frac{x^4}{4!}(-2)$	M1		
	$\approx -\frac{x^2}{2} - \frac{x^4}{12}$	A1	2	CSO throughout part (b). AG
(c)	Limit $= \lim_{x \to 0}\left[\frac{x\ln(1+2x)}{x^2 - \ln\cos x}\right]$			
	$= \lim_{x \to 0}\left[\frac{x(2x - 2x^2 + ..)}{x^2 - \left(-\frac{x^2}{2} - \frac{x^4}{12}..\right)}\right]$	M1		Using earlier expansions
	Limit $= \lim_{x \to 0} \frac{2x^2 - o(x^3)}{1.5x^2 + o(x^4)}$			The notation $o(x^n)$ can be replaced by a term of the form kx^n
	$= \lim_{x \to 0} \frac{2 - o(x)}{1.5 + o(x^2)} = \frac{4}{3}$	M1 A1	3	Need to see stage, division by x^2
	Total		**15**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	$\dfrac{dx}{dt} = e^t \; \{= x\}$	B1		
	$x\dfrac{dy}{dx} = x\dfrac{dy}{dt}\dfrac{dt}{dx}$	M1		Chain rule
	$= x\dfrac{dy}{dt}\dfrac{1}{x} = \dfrac{dy}{dt}$	A1	3	Completion. AG
(ii)	$\dfrac{d^2y}{dt^2} = \dfrac{d}{dt}\left(x\dfrac{dy}{dx}\right) =$			
	$= \dfrac{dx}{dt}\dfrac{dy}{dx} + x\dfrac{d}{dt}\left(\dfrac{dy}{dx}\right)$	M1		Product rule
	….. $= \dfrac{dy}{dt} + x\dfrac{dx}{dt}\dfrac{d}{dx}\left(\dfrac{dy}{dx}\right)$	M1		
	…. $= \dfrac{dy}{dt} + x^2\left(\dfrac{d^2y}{dx^2}\right)$	A1	3	Condone leaving in this form
	$\Rightarrow x^2\dfrac{d^2y}{dx^2} = \dfrac{d^2y}{dt^2} - \dfrac{dy}{dt}$			AG
(b)	$x^2\dfrac{d^2y}{dx^2} - 6x\dfrac{dy}{dx} + 6y = 0$			
	$\Rightarrow \dfrac{d^2y}{dt^2} - 7\dfrac{dy}{dt} + 6y = 0$	M1		Using results in (a) to reach DE of this form
	Auxl eqn $m^2 - 7m + 6 = 0$			
	$(m-6)(m-1) = 0$	m1		PI
	$m = 1$ and 6	A1		PI
	$y = Ae^{6t} + Be^t$	M1		Must be solving the 'correct' DE. (Give M1A0 for $y = Ae^{6x} + Be^x$)
	$y = Ax^6 + Bx$	A1√	5	Ft a minor slip only if previous A0 and all three method marks gained
	Total		**11**	
	TOTAL		**75**	